GROWTH AND DEVELOPMENT

*With Special Reference to Domestic Animals*

VIII. Relation Between Weight Growth and Linear Growth with Special Reference to Dairy Cattle.

/Publication authorized May 1, 1927/
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ABSTRACT

1. When the time rate of tridimensional growth increases at a constant percentage rate, the time rate of linear growth remains approximately constant. When the time rates of tridimensional growth decline at a constant percentage rate, the time rates of linear growth likewise decline at a constant, but higher, percentage rate. The age curve of linear growth may thus be divided into two principal segments (a segment of constant growth rate and a segment of declining growth rate) in the same manner as the age curve of tridimensional growth was divided into two segments (a segment of increasing time rate and a segment of decreasing time rate). The junction in both cases occurs at approximately the same time (puberty).

2. Different linear measurements approach their mature values at different velocities. In the case of dairy cattle, of 21 linear measurements examined, the mature value is approached most rapidly for width of forehead; then length (horns to tail); then height at hips; height at croup; height at withers. At the end of the list come width of hips, circumference of chest and of paunch.

3. The weight increases, roughly, as a power of the linear measurements. The numerical value of this power ranges from 2 to \(5\) depending on the rapidity of approach of the given linear measurement to the mature value. The relation among the linear measurements is, very roughly, linear.

4. The several linear measurements are differently affected by a given kind of undernutrition depending on (1) whether the given measurements approach the mature value rapidly or slowly (the more rapid this approach the less the influence on growth); and (2) whether the measurement is strictly skeletal or whether it includes fleshy growth.

ACKNOWLEDGMENTS

The charts relating to dairy cattle are based on data summarized by A. C. Ragsdale, and the writer in the first bulletin of this series (Missouri Res. Bul. 96). Several figures, and many ideas, discussed in this bulletin have been previously published by A. C. Ragsdale and the writer in Research Bulletins 67 and 80 of this Station.

A portion of the expenses involved in this investigation was paid from a grant from the Committee on Food and Nutrition of the National Research Council. Grateful acknowledgment is made for this cooperation, which was received through the recommendation of Dr. E. B. Forbes, chairman of the Sub-committee on Animal Nutrition.
GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

VIII. The Relation Between Weight Growth and Linear Growth with Special Reference to Dairy Cattle.

SAMUEL BRODY

I. INTRODUCTION: THE COURSE OF LINEAR GROWTH

Before attempting to compare the age curves in weight growth and linear growth, it seems desirable to recall in outline the probable mechanism of growth in the two cases.

Weight growth is, of course, tridimensional growth. In the absence of disturbing factors growth in weight tends to proceed in a geometrical progression: One cell tends to give rise to two cells, two cells to four, four to eight, etc. Such a condition of growth in a geometrical progression (i.e., growth at a constant percentage rate and increasing time rate) has been realized in in vitro systems in which the environmental conditions were kept constant by means of continuous irrigation (Carrel). In the normal course of events, however, the conditions for growth become less favorable with increasing age, with the result that the initial course of increasing time rate of growth is changed, sooner or later, to a course of decreasing time rate of growth. This situation gives to the growth curve an S-shaped form as illustrated in Fig. 1. If the segment of the curve following the inflection is extrapolated on its downward course, it meets the age axis not at zero (conception), but at some later age indicated by $t^*$. The situation is quite different as it concerns linear growth. In this case, the growth we are measuring is terminal growth. Strictly terminal growth cannot, of course, proceed in a geometric progression (increasing time rate) when the factors limiting growth remain constant. When other conditions remain the same, and, consequently, when tridimensional growth proceeds in a geometric progression (constant percentage rate), one would expect linear growth, if such growth is terminal, to proceed in an arithmetical progression (constant time rate). When the favorableness of the physiological environment as it affects growth declines, and when, consequently, the time rate of tridimensional growth declines, then the time rate of linear growth would likewise decline.
As a matter of fact the above considerations, which are entirely general in nature, probably represent the actual state of affairs in growth. In Research Bulletin 98 of this series (cf. Fig. 14 and 15) we have shown that growth in weight in the dairy cow during the first four months of postnatal life takes place at a constant percentage rate. According to the preceding considerations, linear growth during the same period should, therefore, proceed at a constant time rate. That this is approximately the case is shown in Fig. 2.

![Graph showing weight and age relationship.](image)

\[
W = A - Be^{-kt} = 420 - 690 e^{-0.054t}
\]

Jersey Cow

Fig. 1--A typical curve illustrating the course of growth in weight. Circles represent observed values; the smooth curve represents the equation:

\[
W = A - Be^{-kt}
\]

in which \(W\) is the weight at age \(t\), \(A\) is the mature weight, \(B\) a parameter the significance of which is indicated; or

\[
W - A (1 - e^{-k(t-t*)})
\]

in which \(t*\) is the age at which the theoretical curve meets the age axis. The lettering and tracing of this and the following charts were done by R. C. Hase, H. H. Kibler, and J. A. Boden, students in the University of Missouri.

Following the age of 4 months, the time rates of linear growth as well as the time rates of tridimensional growth decline at a constant percentage rate as explained for the latter in Research Bulletins 97 and 101 of this series. Figs. 3a and 3b are demonstrations of this statement as they refer to linear growth in Jersey and Holstein cattle. Fig. 3c represents a similar curve for horses. In the following bulletin (Res. Bul. 104) of this series the same principle will be shown to be applicable to the growth curve of man. This matter has also been discussed by A. C. Ragsdale and the writer in Res. Bull. 80 of this Station.

If the theoretical curves of linear growth are extrapolated to meet the age axis, they approach much nearer to zero time (conception),
than the theoretical curve for growth in weight (Fig. 1) for the reason that the curve of linear growth has no segment of increasing time rate. Several curves in Fig. 3 have been thus extrapolated, including the one for which \( t^* \) comes nearest conception, namely height at withers.

In order to ascertain the generality of the statement that following the major inflection (puberty) the time rate of growth declines by a constant percentage rate as represented by the equation in legend to Fig. 1, this equation was fitted to two sets of data which represent entirely different types of linear growth, namely linear regeneration of the tail.
of the tadpole, and linear growth of the ear in the rabbit. The results are presented in Figs. 4a and 4b.

Fig. 3a.—Linear growth in Jersey cattle. The smooth curves passing through the observed values (circles, etc.) represent equation (1) in legend to Fig. 1. A high value of \( k \) (as for example measurement 9, width of forehead) indicates rapid approach to mature value; i.e., early maturity. A low \( k \) (as e.g., measurement 18, circumference of chest) represents slow approach to maturity, i.e., late maturity. The numbers on the curves refer to the measurements in Table 1.

In the following bulletin (Res. Bul. 104 of this series) it will be shown that the same ideas hold true for the course of linear growth in man.
Fig. 3b—Linear growth for Holstein cattle. For further explanations see legend for Fig. 3a.
Fig. 3c—Linear growth of Percheron horses. (Data by Trowbridge and Chittenden, of Research Bulletin 96.)
Fig. 4a—The course of linear regeneration in the tail of the tadpole *Rana clamitans* is the same as the course of linear skeletal growth in cattle. (Data by M. L. Durbin.)

Fig. 4b—The course of linear growth of the ear of the rabbit is the same as the course of linear skeletal growth in cattle. (Data by Castle and associates.)
II. EQUIVALENCE CHARTS FOR LINEAR GROWTH AND WEIGHT GROWTH

Since linear and tridimensional growth during the self-inhibiting phase of growth may be represented by the same equation (see legend to

Age for Linear Measurements

<table>
<thead>
<tr>
<th>C</th>
<th>B</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>No.11+12+13.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>B</td>
<td>10</td>
<td>20</td>
<td>30</td>
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<td>B</td>
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<tr>
<td>C</td>
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<tr>
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<td>40</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

Fig. 5a—Equivalence chart between growth in weight and linear growth of Jersey cattle prepared according to the method described in Res. Bull. 102 of this series. Note that it requires much less time to reach a given fraction of the mature value for the linear dimensions than for weight. Thus 90 per cent of the mature height at withers (measurement 1) is reached in 18 months after birth, while it requires over 40 months for the body weight to reach the same fraction of the mature weight.
Fig. 1), it follows that it is possible to prepare equivalence charts for the two kinds of growth by the methods described in Research Bulletin 102 of this series. This we have done with the results shown in Figs. 5a to 5c. The curves are numbered, the numbers referring to the measurements given in Table 1 in this bulletin, and to Fig. 11, dairy Section of Research Bulletin 96 of this series.

While, evidently, the curves for growth in weight and for linear growth coincide, the various fractions of the mature size, designated by
**Fig. 5c—Equivalence chart between growth in height and growth in other linear dimensions.**
A, are not reached at the same ages for the several curves. Thus, for the Holstein chart, while 90 per cent of the mature weight is reached at 50 months, the same percentage for height at withers is reached at 19 months; girth of chest at 25 months; width of forehead at 14 months.

These differences may be brought out to greater advantage by plotting weight and linear measurements in terms of percentages of the corresponding mature values on the same age axis as is done in Fig. 6a. The width of forehead is seen to be the first to reach the maximum, or mature, value; then height at withers; finally weight. The body weight
is thus the last to reach its mature value. Perhaps this is the reason that adverse conditions, especially in the later periods of growth, exert such a relatively slight influence on the course of linear growth. Most of the linear growth is completed while growth in weight is still far from the maximum. Fig. 6b shows a similar situation for growth of horses.

Fig. 6b—Growth curves in weight and in linear dimensions of Percheron horses plotted in terms of percentages of the mature Size A. See legend to Fig. 6a for further explanations.

In this connection Fig. 7 will be of interest. It shows the relative percentage of growth in weight and in several linear measurements before and after birth.
Fig. 7—The percentages of growth during the prenatal and postnatal periods of growth with respect to different linear measurements of cattle. See Table I for the measurements corresponding to the numerals. Note that while the birth weight is only 6 per cent of the maximum weight, the width of forehead at birth is nearly 60 per cent and the length of leg nearly 70 per cent of the mature value. Compare with Fig. 6a.

III. CONSTANTS FOR LINEAR GROWTH

While linear and tridimensional growth follow the same course, they differ with respect to the speed (designated by $k$ in our growth equation) of approach to their respective mature values. The numerical values of the velocity constants, $k$; of the mature sizes, $A$; and of the ages at which the theoretical curves meet the age axis, $t^*$, are the constants which define the growth curve. The numerical values of these constants for the curves of linear growth given in Fig. 3 and for the other measurements given in Table 11 and Fig. 10 of Research Bulletin 96 of this series are presented in Table 1. In this table are also given the ages (counted from birth) at which half, three-fourths, and 98 per cent of the mature values, $A$, are reached.
Table 1.—The measurements, the numbers by which the measurements are indicated on the charts, the values of the measurements at maturity (A), the monthly percentage decline in the time rate of growth (100k), the constants B, and # (based on age counted from conception and on the assumption that the period of prenatal growth in dairy cattle is 9.4 months), the ages (from birth) when one half, three fourths and 98 percent of the mature values are reached.

<table>
<thead>
<tr>
<th>Numbers and names of measurements</th>
<th>The numerical values of the measurements at maturity (A)</th>
<th>Monthly percentage decline in growth (100k)</th>
<th>Empirical Constant (Conception) (B)</th>
<th>t# (from Conception)</th>
<th>Age (from birth at)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jersey</td>
<td>Holstein</td>
<td>Jersey</td>
<td>Holstein</td>
<td>Jersey</td>
</tr>
<tr>
<td>cm.</td>
<td>in.</td>
<td>cm.</td>
<td>in.</td>
<td>mos</td>
<td>mos</td>
</tr>
<tr>
<td>1. Height at withers</td>
<td>125.9</td>
<td>94.6</td>
<td>134.7</td>
<td>53.0</td>
<td>9.0</td>
</tr>
<tr>
<td>2. Height at highest point of croup</td>
<td>125.0</td>
<td>94.2</td>
<td>133.0</td>
<td>52.4</td>
<td>11.0</td>
</tr>
<tr>
<td>3. Height at hips points</td>
<td>123.0</td>
<td>128.4</td>
<td>120.0</td>
<td>50.8</td>
<td>12.2</td>
</tr>
<tr>
<td>4. Depth of chest just behind elbow joint</td>
<td>65.3</td>
<td>25.7</td>
<td>69.0</td>
<td>27.2</td>
<td>9.5</td>
</tr>
<tr>
<td>5. Width of chest just behind elbow joint</td>
<td>40.0</td>
<td>15.7</td>
<td>44.4</td>
<td>17.5</td>
<td>8.0</td>
</tr>
<tr>
<td>6. Width of hips (hip joints)</td>
<td>50.0</td>
<td>19.7</td>
<td>55.0</td>
<td>21.7</td>
<td>6.7</td>
</tr>
<tr>
<td>7. Width of loin (center)</td>
<td>35.7</td>
<td>14.1</td>
<td>39.0</td>
<td>15.4</td>
<td>6.5</td>
</tr>
<tr>
<td>8. Length from poll to point of muzzle</td>
<td>49.3</td>
<td>19.4</td>
<td>54.6</td>
<td>21.5</td>
<td>11.5</td>
</tr>
<tr>
<td>9. Width of forehead</td>
<td>19.2</td>
<td>7.5</td>
<td>21.0</td>
<td>8.3</td>
<td>15.0</td>
</tr>
<tr>
<td>10. Circumference of muzzle at opening of mouth</td>
<td>41.0</td>
<td>16.1</td>
<td>45.2</td>
<td>17.8</td>
<td>10.0</td>
</tr>
<tr>
<td>11. Length from horns to base of withers</td>
<td>54.5</td>
<td>21.5</td>
<td>57.5</td>
<td>22.6</td>
<td>10.0</td>
</tr>
<tr>
<td>12. From highest point of withers to a line between hips</td>
<td>86.0</td>
<td>33.9</td>
<td>98.0</td>
<td>38.6</td>
<td>9.5</td>
</tr>
<tr>
<td>13. From a line between hips to tail</td>
<td>109.0</td>
<td>42.9</td>
<td>119.0</td>
<td>46.9</td>
<td>11.5</td>
</tr>
<tr>
<td>14. From point of shoulders to point of hips</td>
<td>155.0</td>
<td>61.0</td>
<td>64.6</td>
<td>8.8</td>
<td>9.2</td>
</tr>
<tr>
<td>15. From point of shoulders to ischium</td>
<td>150.0</td>
<td>48.0</td>
<td>64.6</td>
<td>8.8</td>
<td>9.2</td>
</tr>
<tr>
<td>16. From point of hips to ischium</td>
<td>46.0</td>
<td>18.1</td>
<td>47.0</td>
<td>18.5</td>
<td>9.1</td>
</tr>
<tr>
<td>17. From point of hips directly forward to last rib</td>
<td>35.0</td>
<td>13.8</td>
<td>35.0</td>
<td>13.8</td>
<td>7.6</td>
</tr>
<tr>
<td>18. Heart girth</td>
<td>170.0</td>
<td>66.9</td>
<td>185.0</td>
<td>72.8</td>
<td>8.3</td>
</tr>
<tr>
<td>19. Girth of paunch just behind last rib</td>
<td>200.0</td>
<td>78.7</td>
<td>216.0</td>
<td>85.0</td>
<td>8.6</td>
</tr>
<tr>
<td>20. Smallest circumference of shin bone of fore leg</td>
<td>15.5</td>
<td>6.1</td>
<td>17.8</td>
<td>7.0</td>
<td>11.0</td>
</tr>
<tr>
<td>21. Smallest circumference of shin bone of hind leg</td>
<td>17.2</td>
<td>6.8</td>
<td>19.9</td>
<td>7.8</td>
<td>11.0</td>
</tr>
<tr>
<td>11. - 12 + 13 (Horns to tail)</td>
<td>173 cm</td>
<td>68.1</td>
<td>185 cm</td>
<td>72.8</td>
<td>12.6</td>
</tr>
<tr>
<td>22. Weight</td>
<td>420 lbs</td>
<td>926 lbs</td>
<td>550 lbs</td>
<td>1215 lbs</td>
<td>5.4</td>
</tr>
</tbody>
</table>
IV. THE FUNCTION RELATING GROWTH IN WEIGHT TO GROWTH IN LENGTH

1. The Effect of Adverse Conditions on the Relation Between Growth in Weight to Growth in Length.—A brief discussion of the relative influence of adverse conditions on growth in weight and on linear growth will be instructive in this connection and serve as an introduction to this section.

First Waters and his associates, then Trowbridge, Moulton, and Haigh have found in their investigations in this Station that a quantitative reduction in the customary kind of feed supplied to steers results in a relatively much greater underweight than in underheight of the experimental subjects. These results are interpreted to mean that the "growth impulse" for height is greater than for weight; that is to say, that when there is competition between different organs for a small (i.e., growth-limiting) supply of foods, some organs are more successful in appropriating their share, than are other systems, and the skeletal system is more successful in this respect than the other systems which largely contribute to the body weight.

This interpretation on the basis of difference of "growth impulse" is plausible enough, but one can think of several alternate interpretations. In the first place, the nutrients needed for linear growth may have been relatively more abundant in the given ration than the nutrients needed for gain in weight. In the aforesaid experiments this may have indeed been the case. In the second place, one is not entirely justified in comparing growth in weight with growth in height as it relates to the effect of under-nutrition because at the time the experiments began (weaning age, i.e., about 5 months) less than one-third of the mature weight was reached, whereas nearly three-fourths of the growth in height was completed; in consequence there was not the opportunity to influence growth in height as there was for influencing weight (cf. Fig. 6). Finally, the weight increases as a power (nearly the fourth power) of height, and consequently a small change in height results in a relatively large change in weight under any conditions. This is not a question of "growth impulse" but rather a question of geometry.

The important idea that we should like to emphasize can perhaps be illustrated best by an analysis of the data of Trowbridge, Moulton, and Haigh. The numerical data may be found in the section on Growth of Steers in Research Bulletin, 96 of this series. Two charts of these data (Figs. 8a and 8b) showing the relation between weight and circumference of chest, and weight and height at withers under different nutritive conditions may suffice for the purpose.
Fig. 8a shows that the relation between weight and circumference of chest is not changed so very much by the different conditions of undernutrition (see also the corresponding charts in Res. Bul. 96 of

Heart Girth

Fig. 8a—The relation between body weight and circumference of chest of three groups of Hereford-Shorthorn steers. The three groups fed on widely different rations show approximately the same relationship between weight and height at withers, even though the ages are quite different for a given point on the curve. This seems to indicate that body weight and chest girth are affected by food supply to relatively the same extent. Compare with Fig. 8b. The ages of the animals are given by numerals on the chart.
this series). Weight and girth are apparently affected to nearly the same relative extent by the conditions of undernutrition. The three curves for the three groups of animals come quite close together, although, of course, a given point on the curve represents different ages in the different groups. Thus a full-fed animal at 22 months has the same weight and chest girth as a medium-fed animal at 48 months; a 13-month-old individual of the full-fed group has the weight and girth of a scantily fed animal at 41 months. But the fact is that the curves are not far apart; the nature of the food supply influences the course of growth in weight and in circumference of chest to very nearly the same extent. One reason is that the measure for circumference of chest includes not only the skeletal growth but also the flesh around the chest, and, naturally, underfeeding reduces the amount of this flesh. The circumference of chest increases in size with the increase in body weight. The same may be said with regard to the size of the organs housed in the chest cavity.

The situation is, however, different with respect to height at withers. The height-at-withers measurement does not measure fleshiness. It need not continue to increase with the increase in body weight. The height at withers includes the depth of chest which is no doubt affected by the bulk of the internal organs, but the depth of chest constitutes but a portion of the height-at-withers measurement. The remainder is length of leg which is relatively independent of body weight. So on a priori considerations we may expect the height-at-withers measurement to be much less affected by food supply than the circumference-of-chest measurement. This is clearly illustrated in Fig. 8b.

Fig. 8b shows that for a given height at withers, at the later ages, there are very considerable differences in weight. Thus at a height of 140 cm the weight of the animals in the "poor-growth" group is about 400 kilos; in the "full-fed" group about 800 kilos.

The theoretical conclusion is that such of the measurements which approach their mature values at relatively rapid rates, such as height at withers for example, are but slightly influenced by environmental conditions as compared with the influence of the same condition on weight. On the other hand, such linear measurements as girth of chest or of paunch which may be said to participate in growth of body weight due to depositions of flesh in these regions (that is to say, measurements which approach the mature weight at a relatively slow rate) tend to be influenced by underfeeding to the same relative extent as growth in weight.

The practical lesson of this discussion is this: since height at withers in cattle is but slightly influenced by environmental conditions, therefore at a given age the numerical value of this measurement is practical-
ly a *genetic* index of the size of the animal. To determine the influence of experimental conditions on growth in weight of a given individual, it is only needed to (1) measure the weight and height of the animal; (2) determine the weight for the given height in the animals under normal, that is, control conditions (with the aid of a weight-height curve); (3) find the difference between the weight of the given animal under observation and the weight for the given height from the weight-height curve of
normal animals. The resulting difference is an expression of the effect of the given experimental conditions on the absolute weight of the animal.

This method offers a simple solution to the problem of evaluating the influence of experimental conditions on growth in weight of cattle and like animals.

The relation between weight and height at withers for Holstein and Jersey cattle under “normal” conditions is given in Fig. 10a, and in different form, in Research Bulletin 96 of this series.

While discussing the problem of quantitative evaluation of the nutritive conditions of animals, it seems desirable to mention the pelidisi which von Pirquett employed to measure the nutritional condition of children. This index, in brief, consists of the ratio of the cube root of the body weight (or the cube root of ten times the weight) to the sitting height. This ratio should be near 100 for normal adults. We may, of course, use this index for cattle of a given age and breed after, however, introducing the following slight modification. Let us take the value of this ratio, \( \frac{W^{\frac{1}{3}}}{L} \), (Where \( W \) is weight and \( L \) height at withers) as unity for “normal” animals of a given age. The ratio of \( \frac{W^{\frac{1}{3}}}{L} \) of the given animal, to \( \frac{W^{\frac{1}{3}}}{L} \) of the standard, or “normal”, animal when multiplied by 100 gives, then, the percentage of the “normal” nutritive condition of the animal under consideration. Instead of using \( \frac{1}{3} \) as the exponent of weight, we may use another value, such as \( \frac{1}{3.81} = .262 \) as the exponent which gives an approximately constant value for (Holsteins) of all ages (we shall presently show that the ratio of \( \frac{W}{L^{3.81}} \) is approximately constant for Holsteins of all ages).

2. The Function Relating Tridimensional and Linear Growth.—The form, or conformation, of the animal changes with age as shown in Figs. 7 and 9. The specific gravity of the animal likewise changes with age—but this may be ignored for the present purpose. If the form did not change with age, i. e., if the animal were cubical and continued in the same form during the period of its growth, then the relation between linear growth and weight growth could be presented by the equation

\[ W = CL^3 \]

(1)
in which \( W \) is the weight for the linear measurement \( L \). \( C \) is a constant.

But growth and development imply change in form. Hence the above relationship cannot be true. What we may say is that weight
Fig. 9—The change of form with age in dairy cattle (compare to figures in Res. Bul. 67 of this Station).
tends to increase probably as some constant power \( n \) of the linear measurement \( L \), that is \[ W = CL^n \] (2)

It may be instructive to attempt to fit equation (2) to a series of data. The simplest manner of fitting this equation is to "rectify" the equation by taking logarithms

\[ \log W = \log C + n \log L. \]

That is to say, plotting the logarithms of weight, \( W \) against the logarithms of the linear measurements, \( L \), should give a straight line of slope \( n \) and intercept \( \log C \).

Equation (2) has been fitted by this method to nine sets of data (Fig. 10a, 10b, 10c, etc.). The value of the exponent, \( n \), ranges in

![Graph showing the relationship between weight and height at withers of growing Jersey and Holstein females. The circles represent observed values, the smooth curves represent the equations shown on the chart. B represents the values at birth, the numerals represent ages in months counted from birth. The following seven charts in this series represent the same groups of animals and they are plotted in the same manner. The numerical data are given in Research Bulletin 96 of this series.](image-url)
value from 2 to nearly 5 for the different linear measurements. The agreement between observed values and the values computed from equation (2) are, in general, fair, with, however, many considerable deviations in certain regions of the curve. In some cases (Figs. 11a to 11c), the equation could not be fitted at all satisfactorily.

The reason for at least some of these deviations may be found first, in the fundamental difference in the nature of linear and tridimensional growth preceding five months of age as previously explained; second, in the change of form with increasing age. (For striking changes in form see Figs. 7 and 9.) In the case of the dairy cows under consideration, there are also the additional disturbing factors of gestation
and lactation. During gestation the animals, of course, increase in weight; during lactation of these heavy-milking heifers, there may be an actual loss, or at any rate decrease of growth, in body weight. The effect of these factors on linear growth, on the other hand, is negligible. The results of these cyclic fluctuations are illustrated in Fig. 12.

Fig. 10c—The relation between weight and height at hip points.
Fig. 10d—The relation between weight and the length of body from withers to hips.
Fig. 10c—The relation between weight and distance from shoulder to ischium.
Length from Point of Shoulder to Point of Hip

Fig. 10f—The relation between weight and length from shoulder to hip.
Fig. 10g—The relation between weight and length from hip to ischium.
Fig. 10h—The relation between weight and width of hips.
Fig. 10i.—The relation between weight and height at withers and weight and circumference of chest of Percheron horses. Note that the agreement between observed and computed values is as good for horses as for cattle and that the numerical values of the exponents are not very different for the two species. Incidentally, the females at birth have approximately the same weight but higher values for height at withers and circumference of chest than males.
Fig. 11a—The relation between weight and width of chest. The distribution of the data points is so irregular that no attempt was made to fit equation (2) to them.
Fig. 11b—The relation between weight and depth of chest.
Fig. 11c—The relation between weight and circumference of shin bone.
Fig. 12—The ratios of weight to the several linear measurements which were raised to the powers given in Fig. 10; that is, the values of $C$ in the equation

$$C = \frac{W}{L^n}$$

in which $W$ is the weight for the linear measurement $L$. The values of $n$ are given in Figs. 10a, 10b, and 10h. See text for interpretation of the fluctuations.
V. THE FUNCTION RELATING LINEAL MEASUREMENTS WITH EACH OTHER

Scammon and his co-workers found a linear relation between various linear measurements of man during prenatal growth. These results of Scammon suggested the desirability of ascertaining whether the relation between various linear measurements in dairy cattle during the period of growth is also linear.

Fig. 13 shows that the relation between various linear measurements and height at withers is not far from linear; but it cannot be said to be exactly linear. It is only approximately, or rather roughly, linear. One would hardly expect on a priori considerations for the relations to be strictly linear, since the speed of approach \((k)\) to the mature values is quite different for different linear measurements. Width of forehead, for example, practically ceases growing long before width of hips or girth of paunch. One would, more likely, be expected to assume the relations to be logarithmic. However, the relation between logarithmic and linear equations is often very close, and they are, of course, identical when the exponent, \(n\), in the power equation is unity.

<table>
<thead>
<tr>
<th>Measurement No.</th>
<th>Value of (a)</th>
<th>Value of (b)</th>
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<tbody>
<tr>
<td>2</td>
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<td>+9.4</td>
</tr>
<tr>
<td>3</td>
<td>.863</td>
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<tr>
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<tr>
<td>12</td>
<td>.813</td>
<td>-19.0</td>
</tr>
</tbody>
</table>

Fig. 14 shows how the percentages of each of the several linear measurements to the corresponding heights at withers change with age. Other relationship between linear measurements are given in Research Bulletin 67 of this station.
Fig. 13—The relation between some of the linear measurements and height at withers of Jersey and Holstein cattle. The relation is very roughly linear; that is, the relations can be represented by the equation

\[ L = aH + b \]

in which \( L \) is any linear measurement, \( H \) is the corresponding height at withers, \( a \) is the slope of the line and \( b \) is the intercept (the value of \( L \) when \( H = 0 \)).

The numerals on the curves refer to the measurements given in Table I. The numerical values of \( a \) and \( b \) for the several measurements (Holstein and Jersey combined) are, roughly, as shown in the tabulation on the opposite page.
Fig. 14—The linear measurements indicated in the chart plotted in terms of percentages of the heights at withers for the corresponding ages. (For fuller descriptions of the measurements see Table 1.)