METHOD FOR OBTAINING THE TREES OF A V VERTEX COMPLETE GRAPH FROM THE TREES OF A V-I COMPLETE GRAPH

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Reprinted from

Matrix and Tensor Quarterly
Volume 15, Number 3, March 1965

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Method for Obtaining the Trees of a \( v \) Vertex Complete Graph from the Trees of a \( v-1 \) Vertex Complete Graph

A method\(^1\),\(^2\) is described for obtaining the trees of a \( v \) vertex complete graph through an iterative process. The iterative process is as follows, first find all the trees of a 3 vertex complete graph, then by an appropriate substitution the trees of a 4 vertex complete graph can be found. Continuing in this manner the trees of a \( v \) vertex complete graph can be found from the trees of a \( v-1 \) vertex complete graph.

This method follows from inspection of Fig. 1. Each subgraph is a quasi-complete graph, i.e., a complete graph with some parallel edges, the trees of these quasi-complete graphs will produce the trees of the next order higher complete graph with an appropriate edge included in each of the 4 sets of trees. This is a result of separating the trees of complete graph of 5 vertices into 4 distinct subsets in the following way. Remove edge \((1,5)\) and let vertices 1 and 5 coalesce, the trees of this resultant graph contain all trees (when edge \((1,5)\) is included in each tree of the resultant graph) which contain edge \((1,5)\). Remove edge \((1,5)\) from the complete graph of 5 vertices. Now remove edge \((2,5)\) and let vertices 2 and 5 coalesce, the resultant graph contains all the trees of the original complete graph, which contain edge \((2,5)\) but not edge \((1,5)\), again when edge \((2,5)\) is included. Continue in this manner until edge \((4,5)\) is reached. The first set contains all the trees which have edge \((1,5)\), the second set consists of those trees which do not have edge \((1,5)\), but do have edge \((2,5)\). This process is continued until the last set is reached, this set contains edge \((4,5)\) but not edges \((1,5), (2,5), (3,5)\). This development separates the trees of a complete graph into 4 distinct sets, i.e., given any tree \( t \), where \( t \) is a member of the set of all trees of the complete graph of 5 vertices, \( t \) can be found in one and only one of the 4 distinct sets, as developed above.

Before stating the rule for finding the trees of a \( v \) vertex complete graph from the trees of a \( v-1 \) vertex complete graph, an example is given for clarity. Assume the trees of a 4 vertex complete graph are known. Fig. 1 shows the trees of the 5 vertex complete graph graphically. The trees which contain edge \((1,5)\) are the trees of a 4 vertex complete graph which has 3 parallel edges. The edges between vertices 1 and 2 are \((1,2)\) and \((2,5)\), the edges between vertices 1 and 3 are \((1,3)\) and \((3,5)\) and the edges between vertices 1 and 4 are \((1,4)\) and \((4,5)\).

Therefore the trees of the first set are, \((1,5)UT\) (4 vertex complete graph), with edge \((1,2)\) replaced by \((1,2) + (2,5)\), \((1,3)\) by \((1,3) + (3,5)\) and \((1,4)\) by \((1,4) + (4,5)\). The trees of the second set are, \((2,5)UT\) (4 vertex complete graph), with edge \((2,3)\) replaced by \((2,3) + (3,5)\) and \((2,4)\) by \((2,4) + (4,5)\). The trees of the third set are, \((3,5)UT\) (4 vertex complete graph), with edge \((3,4)\) replaced by \((3,4) + (4,5)\). The trees of the final set are, \((4,5)UT\) (4 vertex complete graph). If the trees of the four quasi-complete graphs are collected and each set has edge \((1,5), (2,5), (3,5)\) or \((4,5)\) included as noted, this set of trees will comprise the trees of a 5 vertex complete graph.

This was easily generalized so that the trees of a \( v \) vertex complete graph can be found from the trees of a \( v-1 \) vertex complete graph. Assume the trees of a \( v-1 \) vertex complete graph are known, denote these trees by \( T_{v-1} \), the trees of a \( v \) vertex complete graph are denoted by \( T_v \). The rule for finding \( T_v \) from \( T_{v-1} \) is,
\[ T_5 = T \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix} = (1,5) \begin{bmatrix} 2 & 4 & 3 \end{bmatrix} \]

\[ + (2,5) \begin{bmatrix} 1 & 2,5 & 1 & 4 \end{bmatrix} + (3,5) \begin{bmatrix} 2 & 3,5 & 4 \end{bmatrix} \]

\[ + (4,5) \begin{bmatrix} 2 & 4,5 & 3 \end{bmatrix} \]

\[ T_4 = T \begin{bmatrix} 1 & 2 & 3 & 4,5 \end{bmatrix} \]

\[ T \begin{bmatrix} \end{bmatrix} \equiv \text{Trees of} \]

Fig. 1
\[ T_v = (1, v)UTv^{-1}, \quad \text{with} \quad \begin{cases} (1, 2) = (1, 2) + (2, v) \\ (1, 3) = (1, 3) + (3, v) \end{cases} \]

\[ + (2, v)UTv^{-1}, \quad \text{with} \quad \begin{cases} (2, 3) = (2, 3) + (3, v) \\ (2, 4) = (2, 4) + (4, v) \end{cases} \]

\[ + \ldots + \]

\[ + (v-2, v)UTv^{-1}, \quad \text{with} \quad \begin{cases} (v-2, v-1) = (v-2, v-1) + (v-1, v) \end{cases} \]

\[ + (v-1, v)UTv^{-1}, \]
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